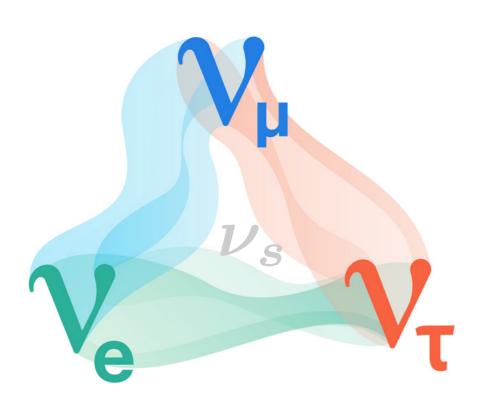
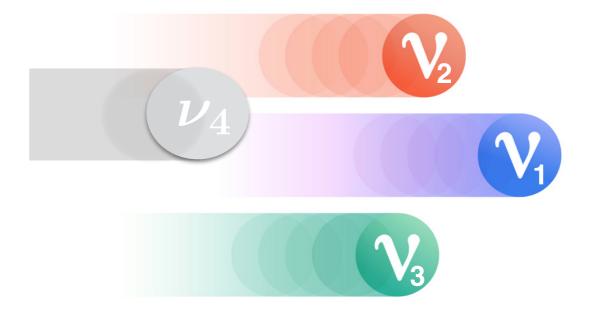




" Δm^2_{21} Measurements and Tensions"



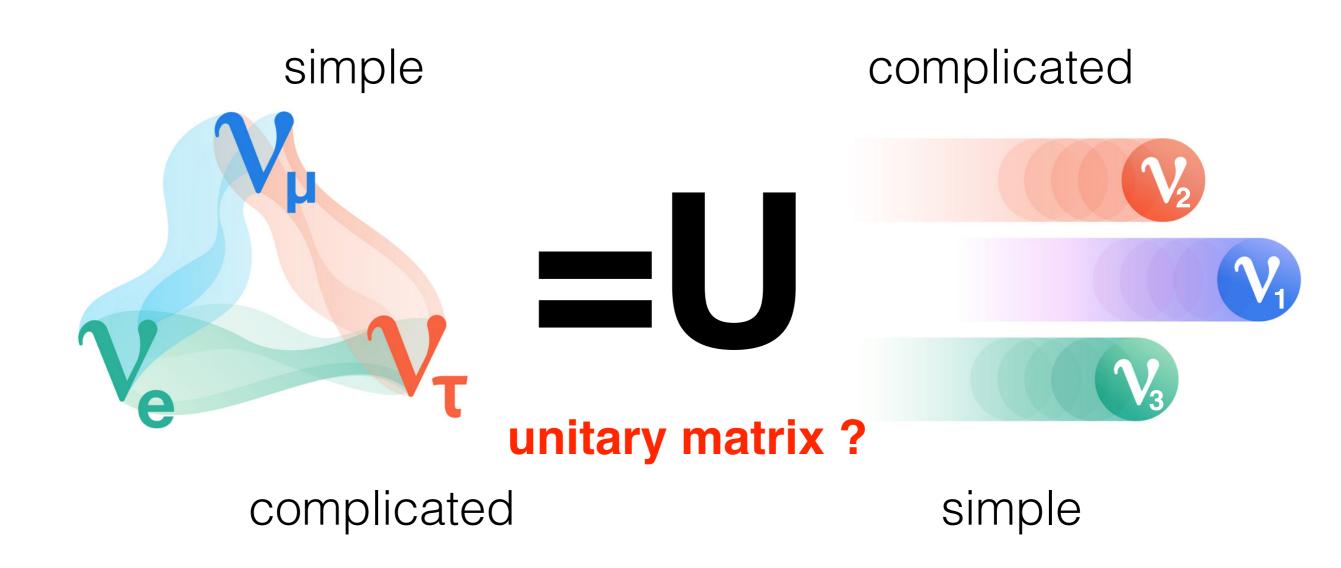
Stephen Parke Fermilab







Interactions:



Propagation:

masses?



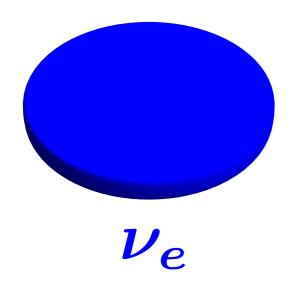


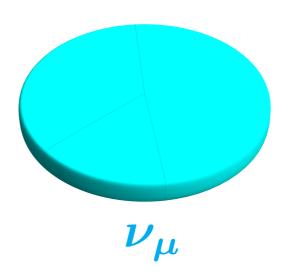
Neutrino Flavor or Interaction States:

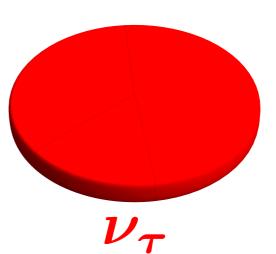
$$W^+
ightarrow e^+
u_e$$

$$W^+ o \mu^+
u_\mu$$

$$W^+ o au^+
u_ au$$







provided $L/E \ll 0.5 \mathrm{~km/MeV} = 500 \mathrm{~km/GeV}$!!!

 ~ 1 picosecond in Neutrino rest frame !!!

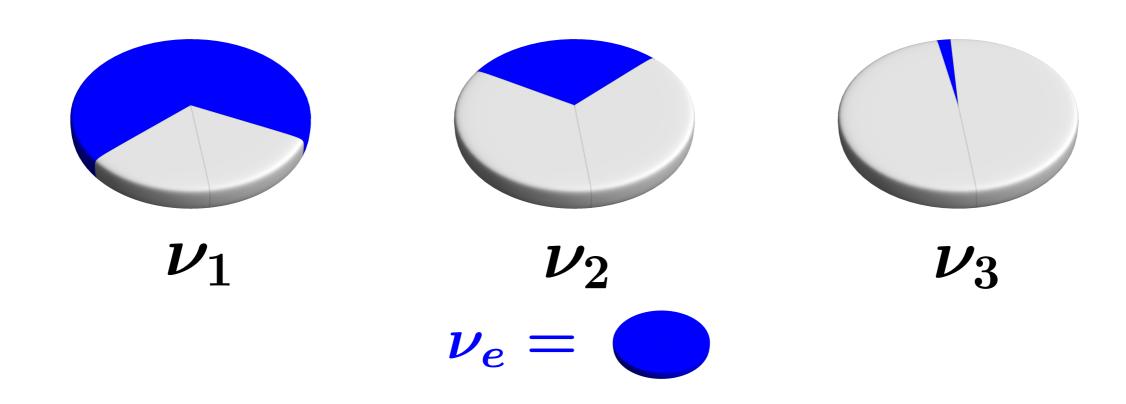


Stephen Parke



Neutrino Mass EigenStates or Propagation States:

Propagator $u_j o
u_k = \delta_{jk} \, e^{-i \, \left(rac{m_j^2 L}{2 E
u}
ight)}$



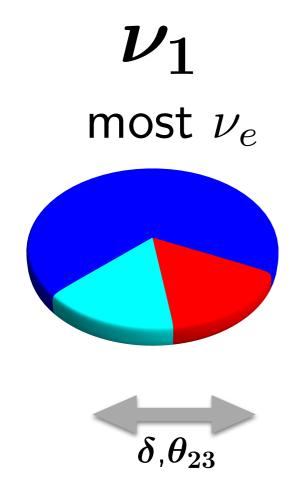
$$\Delta m^2_{21} \sim 7.5 imes 10^{-5} ext{ eV}^2$$

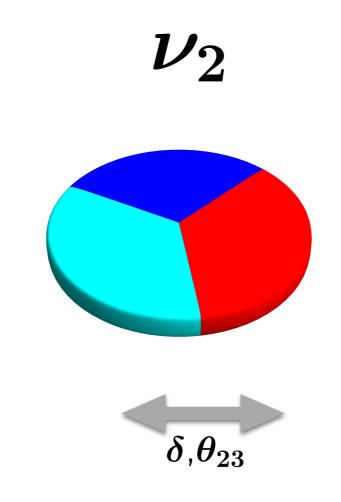
$$|\Delta m^2_{31}| pprox |\Delta m^2_{32}| \sim 2.5 imes 10^{-3} \; ext{eV}^2$$

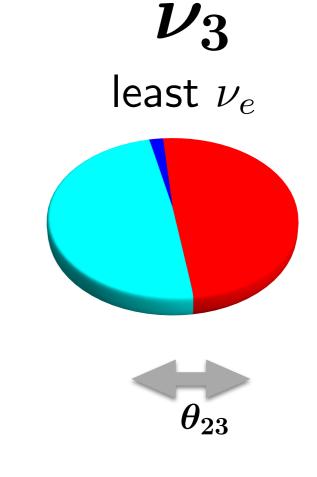


Neutrino Mass EigenStates or Propagation States:









$$\nu_e =$$

Solar Exp, SNO KamiLAND Daya Bay, RENO, ...

$$u_{\mu} =$$
SuperK, K2K, T2K
MINOS, NOvA
ICECUBE

$$u_{ au} = \mathbf{U}$$
Unitarity
SK, Opera
ICECUBE?

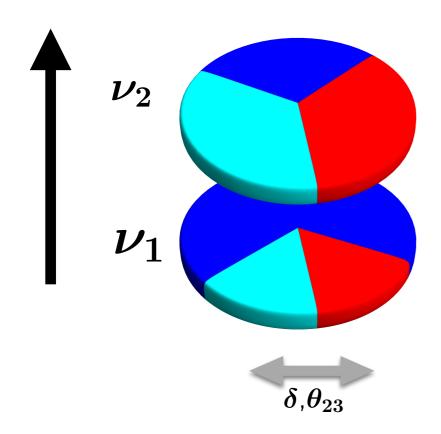


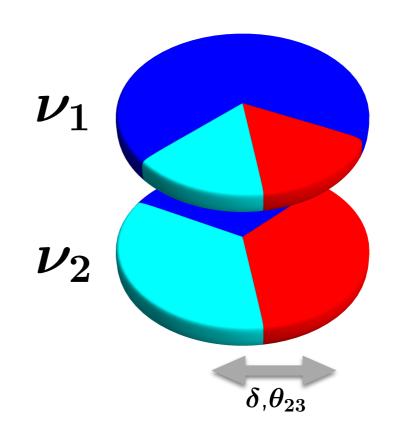


ν_1 , ν_2 Mass Ordering:

-solar mass ordering

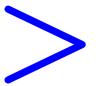
mass





$$|\Delta m_{21}^2| = |m_2^2 - m_1^2| = 7.5 \times 10^{-5} \text{ eV}^2$$
 $L/E = 15 \text{ km/MeV} = 15,000 \text{ km/GeV}$

 $SNO m_2 > m_1$



$$\nu_e =$$

$$u_{\mu} = igcolum_{\mu}$$

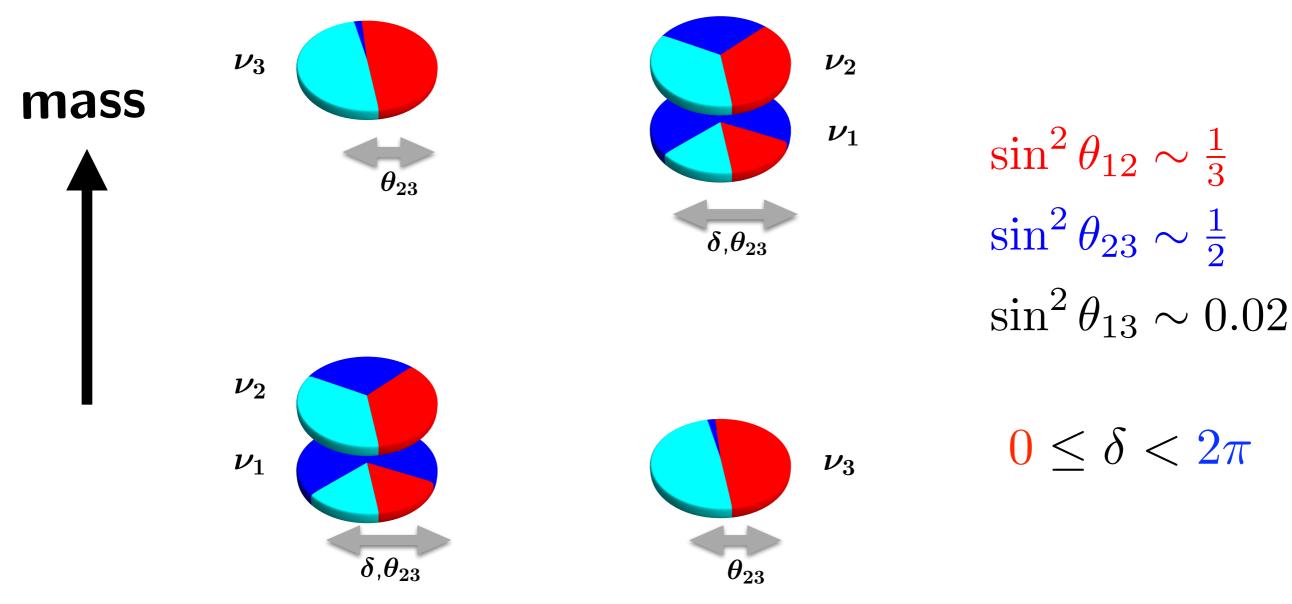
$$u_{\tau} = \bigcirc$$







-atmospheric mass ordering



$$|\Delta m_{31}^2| = |m_3^2 - m_1^2| = 2.5 \times 10^{-3} \text{ eV}^2$$
 $L/E = 0.5 \text{ km/MeV} = 500 \text{ km/GeV}$

Unknown: NOVA, JUNO, ICECUBE, DUNE, T2HKK....

$$\nu_e =$$

$$u_{\mu} = \bigcirc$$

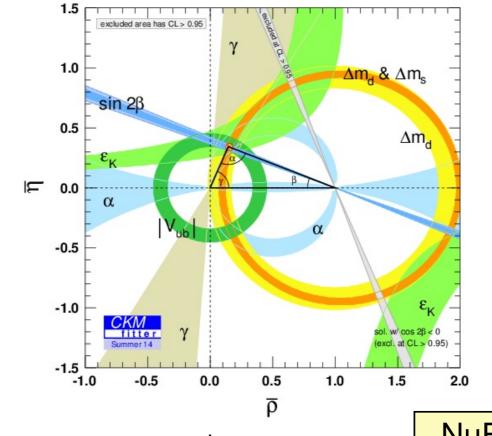
$$\nu_{ au} = lacksquare$$



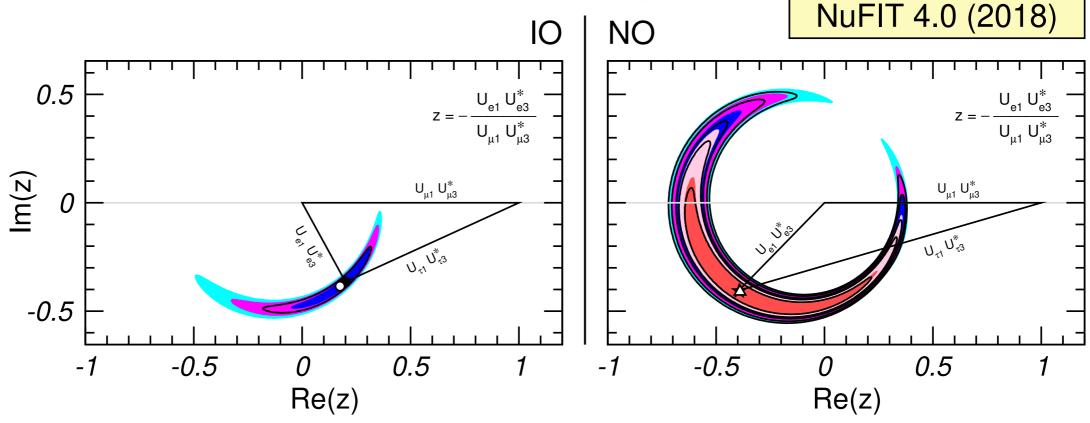


quarks

Unitarity NOT assumed



neutrinos

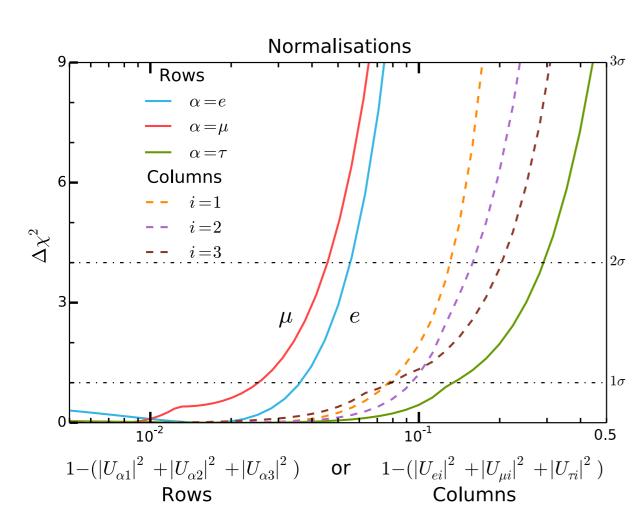


Unitarity *Is* assumed

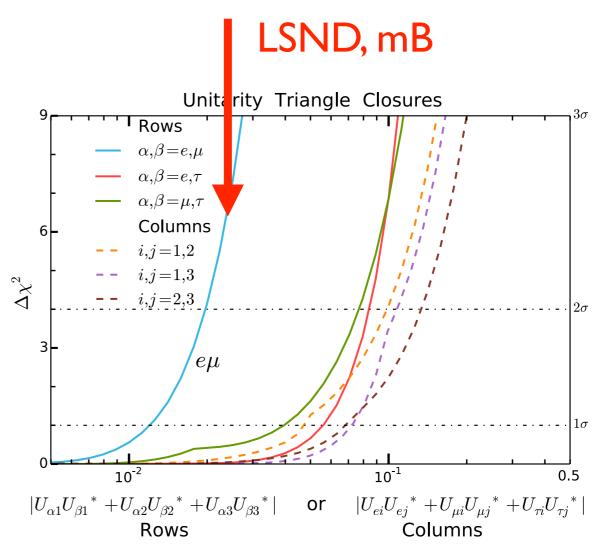


Unitarity ???





Ross-Lonergan+ SP 1508.05095



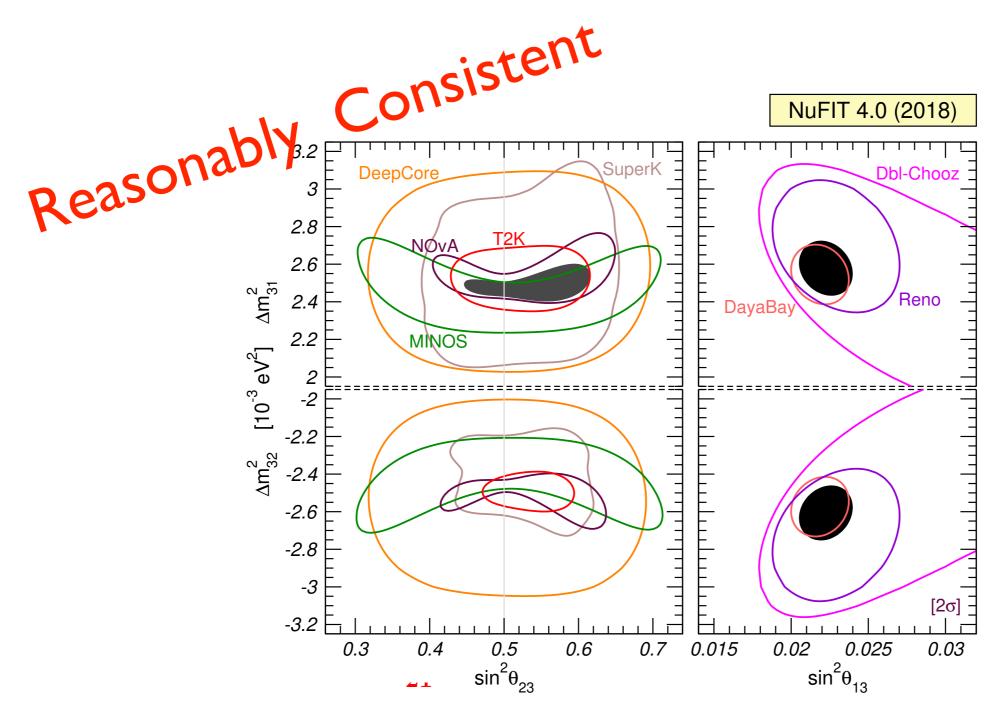
6 row/column plus 6 triangle conditions

2 row and 1 triangle, independent of $\,
u_{ au}$





Δm^2_{atm} v $\sin^2 heta_{23}$ $(\sin^2 heta_{13})$ consistency ?



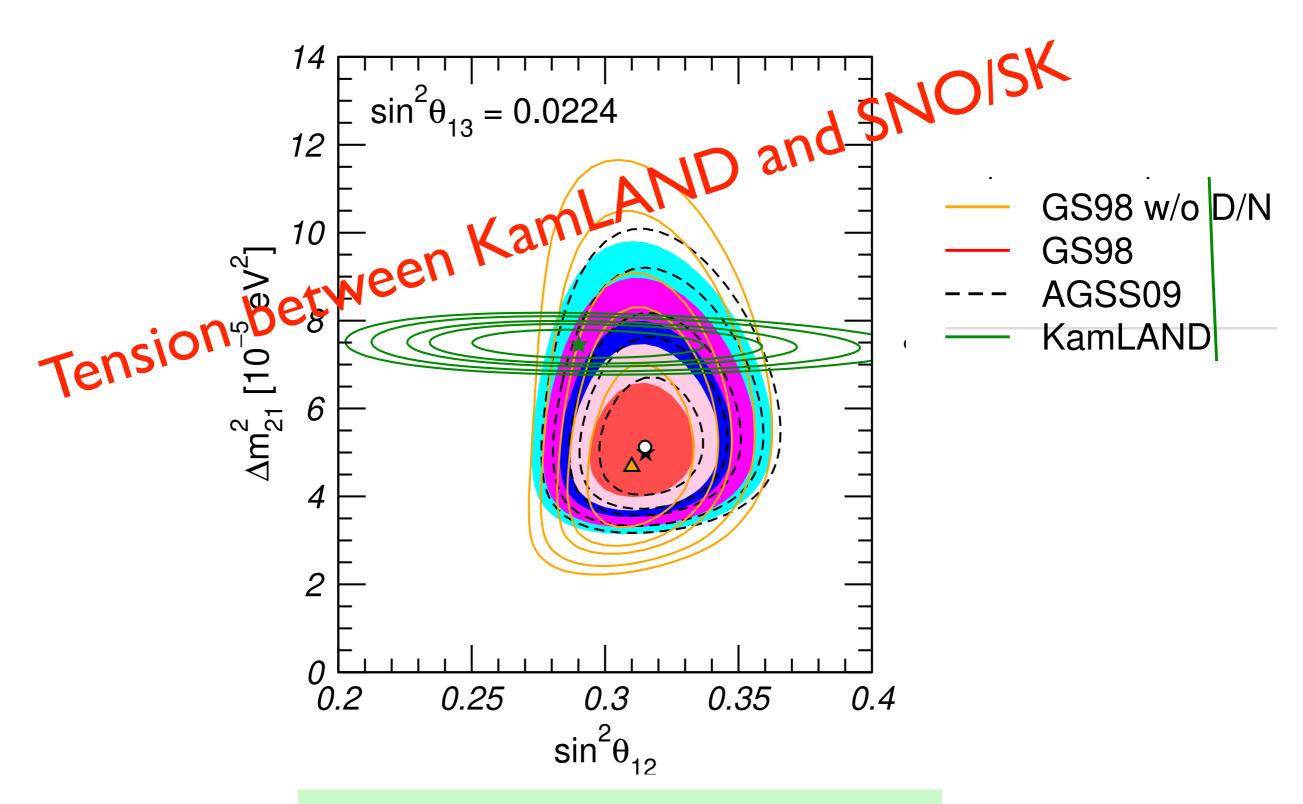
$$|U_{\mu 3}|^2 = c_{13}^2 \sin^2 heta_{23}$$

$$4|U_{\mu 3}|^2(1-|U_{\mu 3}|^2)$$









1σ, 90%, 2σ, 99%, 3σ CL for 2 dof

 Π





Why do we care about

$$\Delta m_{21}^2$$





CP Violation:

At oscillation maximum in vacuum:

$$P(\bar{\nu}_{\mu} \to \bar{\nu}_{e}) - P(\nu_{\mu} \to \nu_{e}) \approx \pi J \left(\frac{\Delta m_{21}^{2}}{\Delta m_{31}^{2}}\right)$$

where J is Jarlskog Invariant (1985):

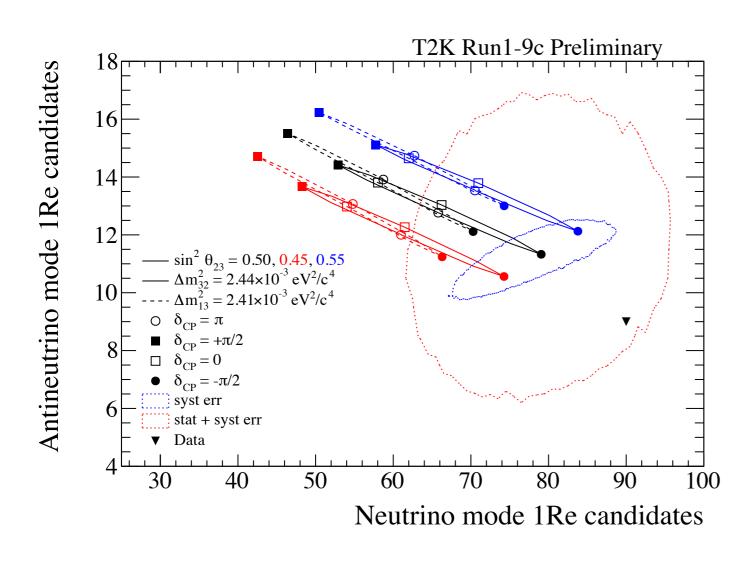


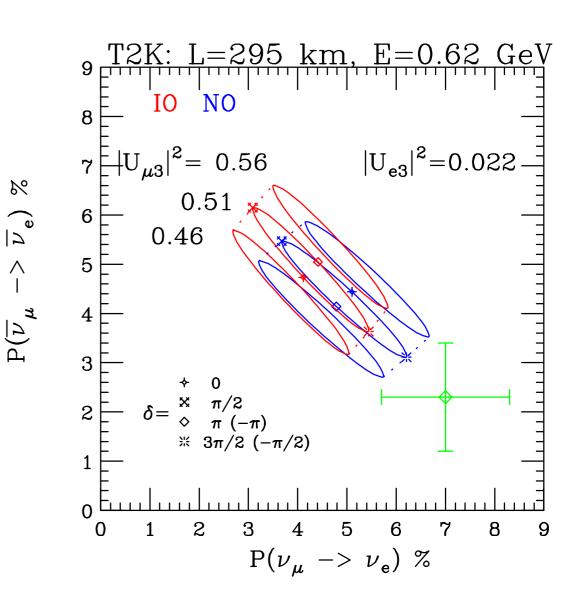
$$J = \sin 2\theta_{12} \sin 2\theta_{13} \cos \theta_{13} \sin 2\theta_{23} \sin \delta \approx 0.3 \sin \delta$$



T2K





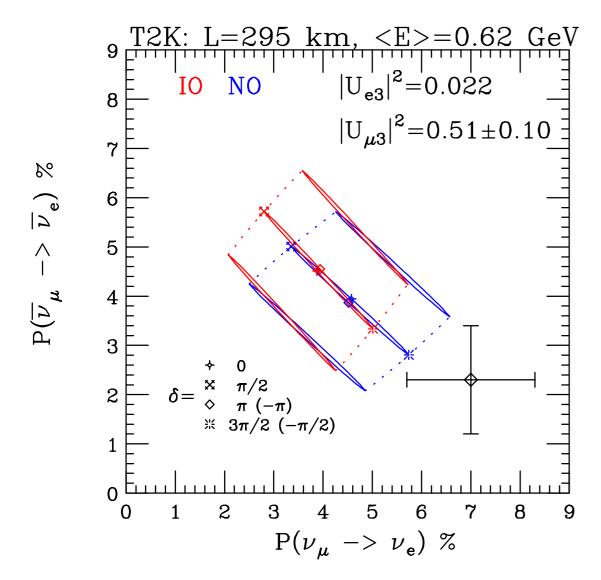


bi-event:

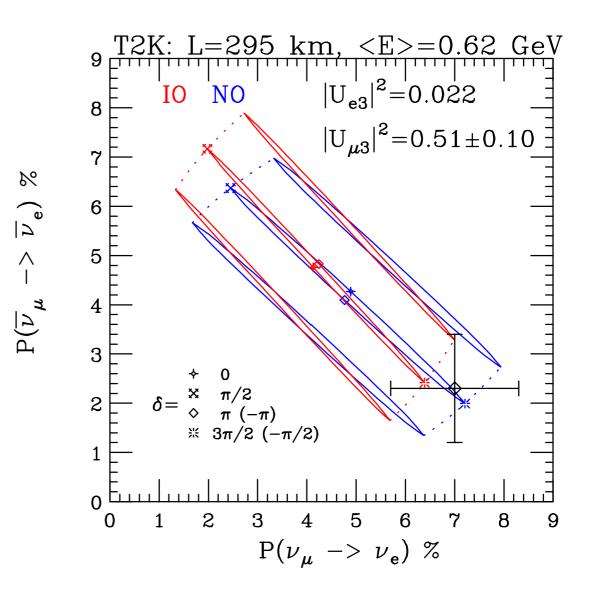
bi-probability:







 $\begin{array}{c} {\rm KamLAND} \\ \Delta m_{21}^2 \end{array}$



2 x KamLAND Δm_{21}^2





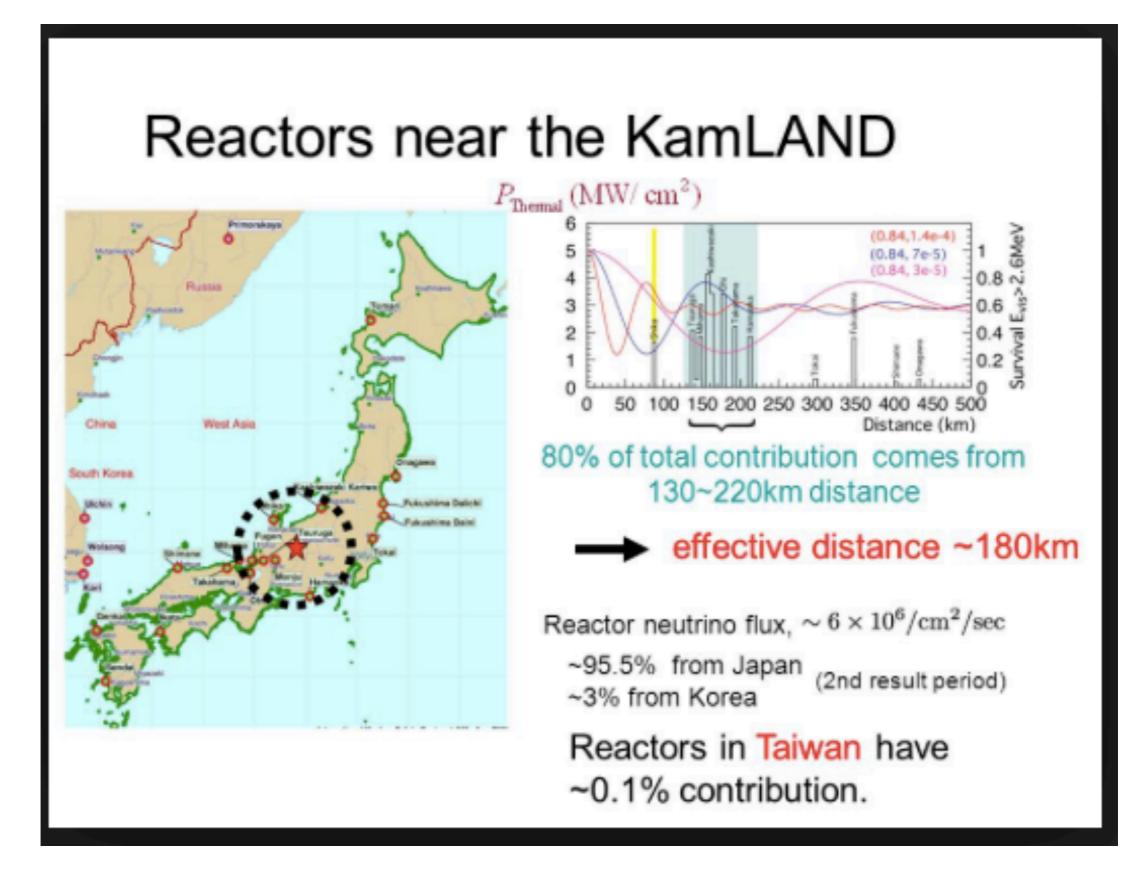
How do we measure

$$\Delta m_{21}^2$$



KamLAND:



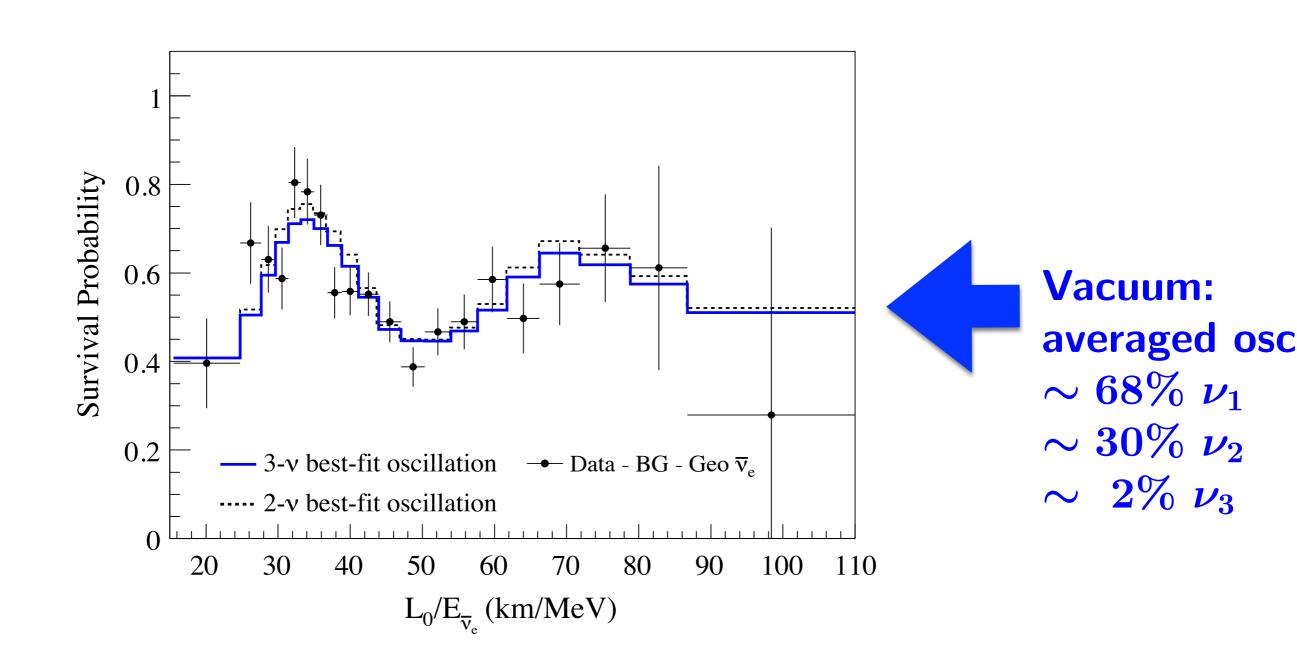


17





KamLAND:

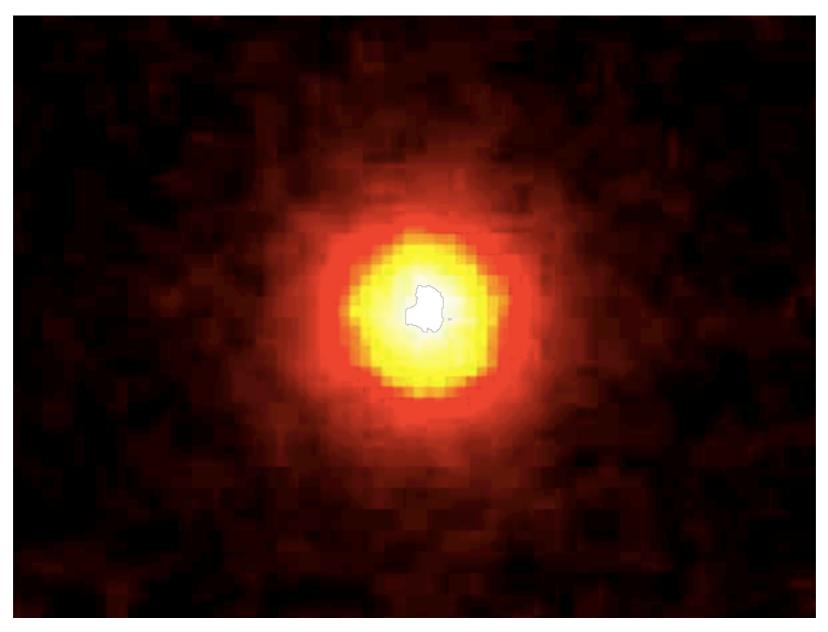


$$\Delta m_{21}^2 = 7.50^{+0.20}_{-0.20} \times 10^{-5} \text{ eV}^2,$$



SuperK





$$\nu$$
? + $e \rightarrow \nu$ + e

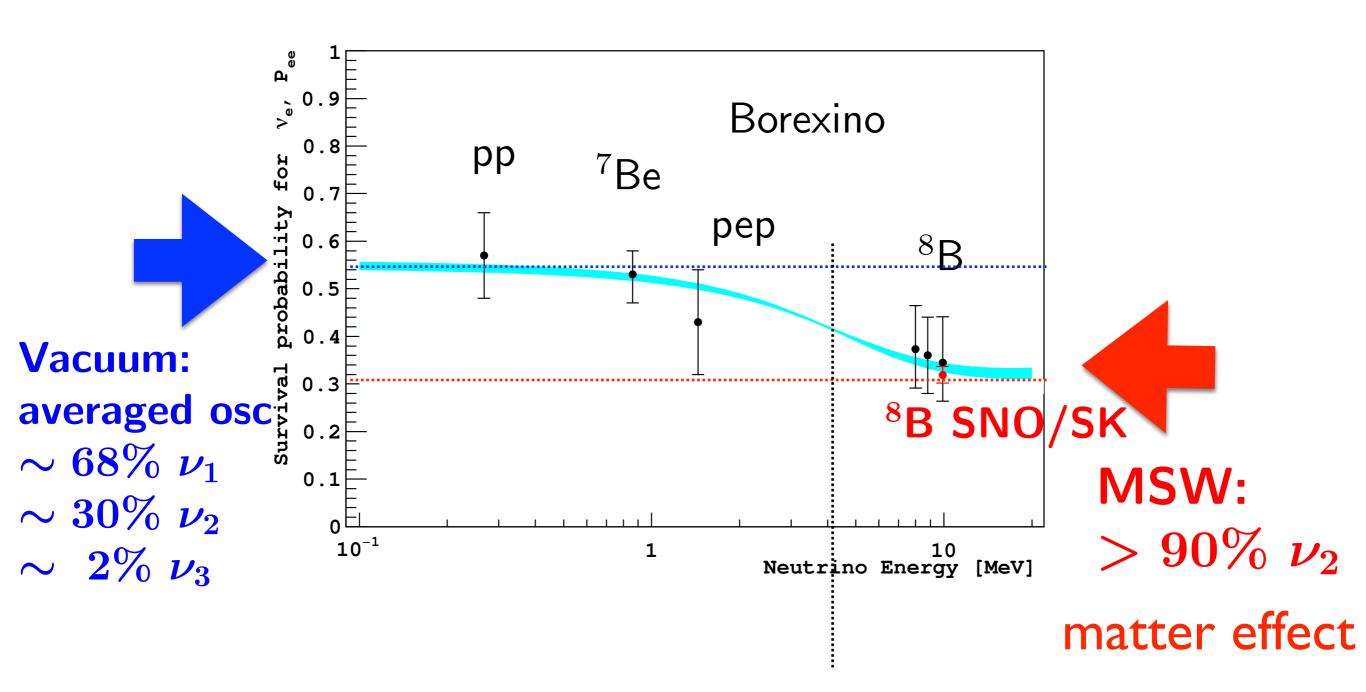
Which type of Neutrino dominates this image?

3/7/2019





Solar Neutrinos:



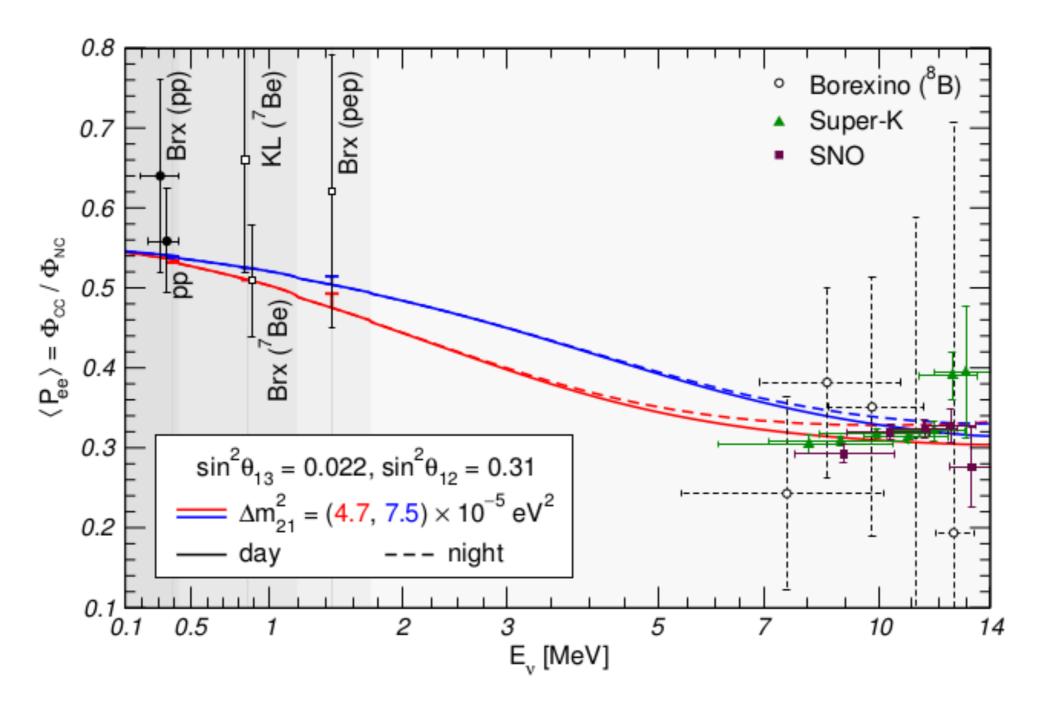
$$E_{
u} = (\,\#\,) \Delta m^2_{21} \cos 2 heta_{12}/(\cos^2 heta_{13} 2\sqrt{2} G_F N_e)$$

Large Δm^2_{21} implies large $E_{
u}$ at transition between Vac. and Matter dominated







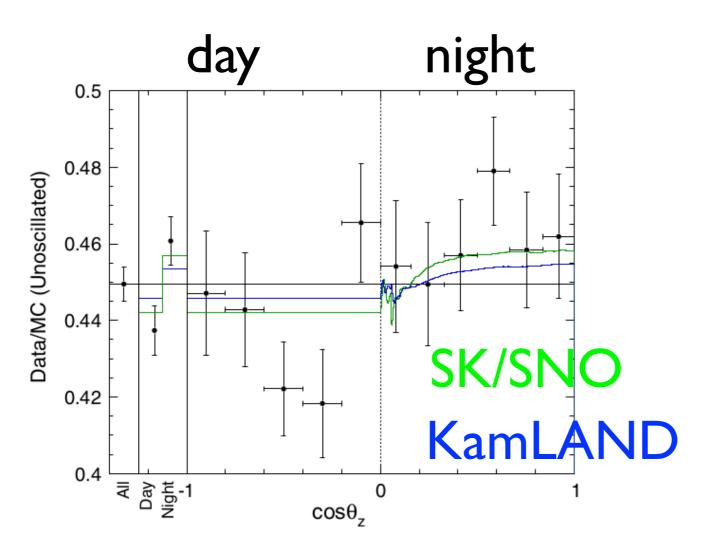


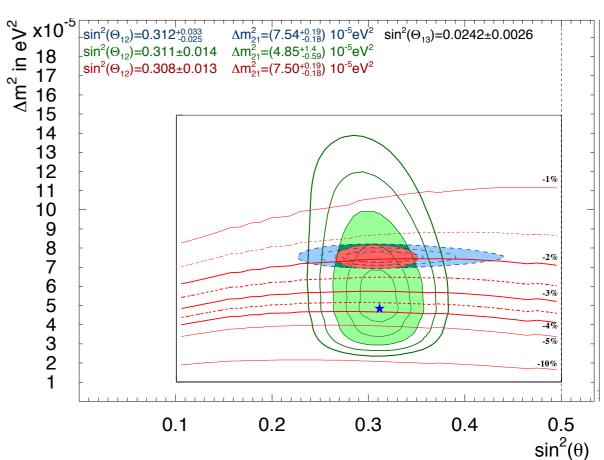
Eur.Phys.J. A52 (2016) no.4, 87



D/N asymmetry







Phys. Rev. D94, 052010 (2016)

$$(D-N)/(D+N) = (\#)(\cos^2 heta_{13}2\sqrt{2}G_FN_e^\oplus)/\Delta m_{21}^2\cos2 heta_{12}$$

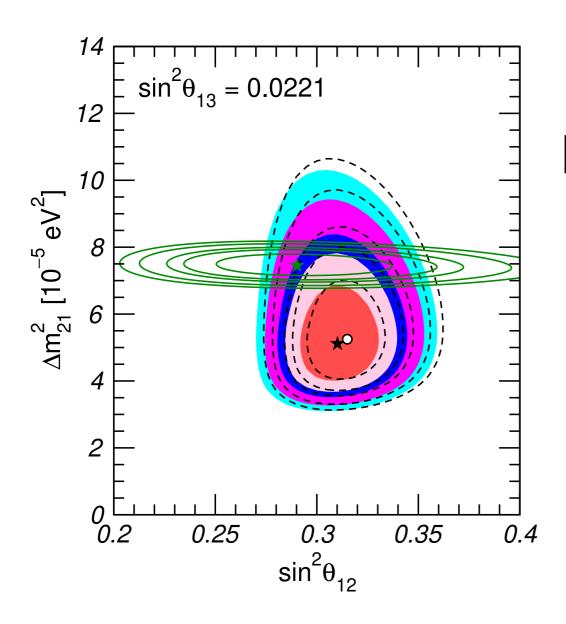
Smaller Δm_{21}^2 implies large D/N Asym.





Tension between KamLAND and SNO/SK

Nu-fit



KamLAND

$$\Delta m_{21}^2 = 7.50^{+0.20}_{-0.20} \times 10^{-5} \text{ eV}^2,$$

SK/SNO

$$\Delta m_{21}^2 = 5.1_{-1.0}^{+1.3} \times 10^{-5} \text{ eV}^2,$$



Scalar NSI



Ge + SP 1812.08376



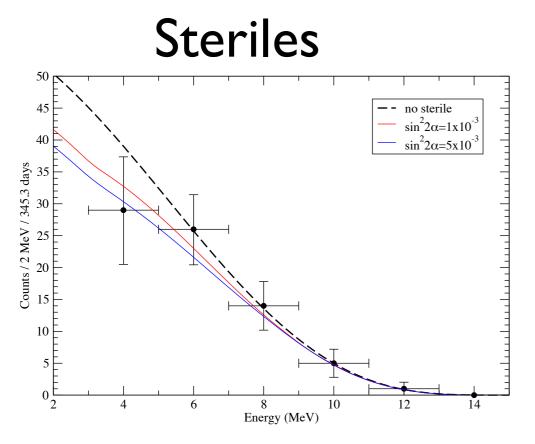


Figure 12: Prediction for B—neutrino spectrum at Borexino versus with experimental data [16]. The neutrino parameters and solar model are the same as in fig. 8.

de Holanda + Smirnov 1012.5627

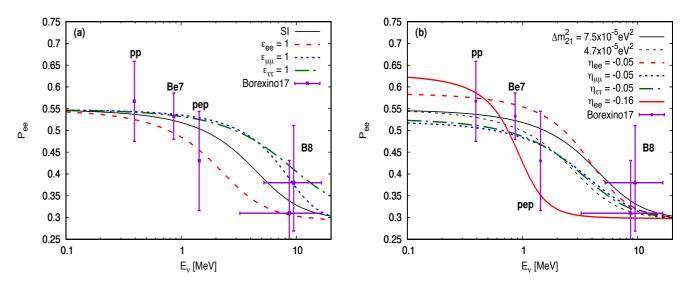
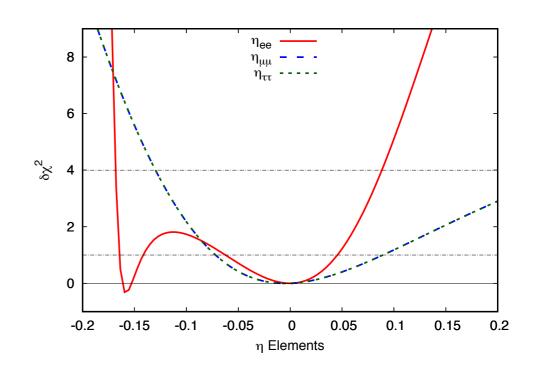


FIG. 2. The solar neutrino convertion probabilities with (a) the vector and (b) the scalar NSIs, together with the Borexino measurement [39] of the pp, ⁷Be, and pep fluxes.







JUNO circa 2025



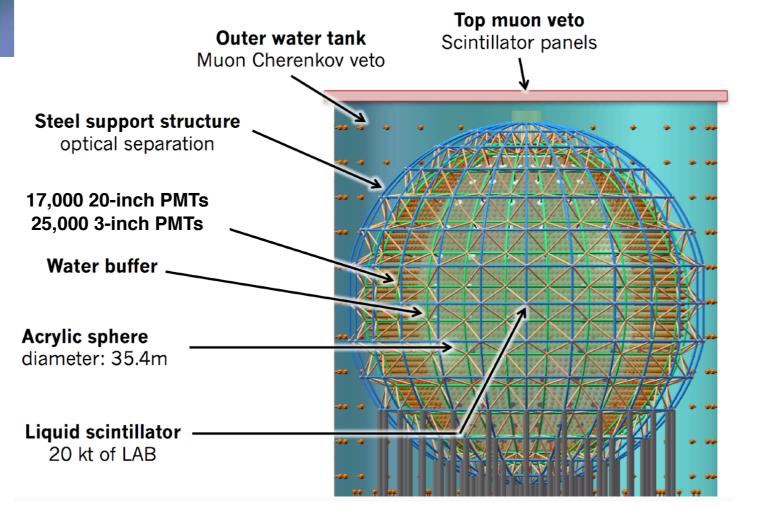


Taishan N

JUNO

LS Detectors Daya Bay Borexino KamLAND JUNO

Target Mass 20 t x 8 300 t 1 kt 20 kt



Similar in concept to previous LS experiments, but much LARGER

In fact, JUNO will be the largest liquid scintillator (LS) detector so far in history!

53 km

Yangjiang NPP





JUNO precision ~2025

$$\sin^2 \theta_{12}$$
, Δm_{21}^2 and $|\Delta m_{ee}^2|$

0.5%

	Nominal	+ B2B (1%)	+ BG	+ EL (1%)	+ NL (1%)
$\sin^2 \theta_{12}$	0.54%	0.60%	0.62%	0.64%	0.67%
Δm_{21}^2	0.24%	0.27%	0.29%	0.44%	0.59%
$ \Delta m_{ee}^2 $	0.27%	0.31%	0.31%	0.35%	0.44%

Table 3-2: Precision of $\sin^2 \theta_{12}$, Δm_{21}^2 and $|\Delta m_{ee}^2|$ from the nominal setup to those including additional systematic uncertainties. The systematics are added one by one from left to right.

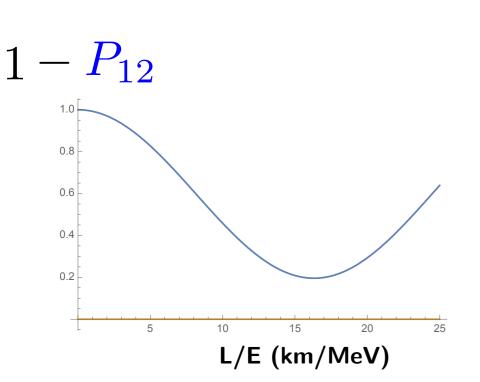
$$\Delta m_{ee}^2(\text{NPZ}) \equiv \cos^2 \theta_{12} \Delta m_{31}^2 + \sin^2 \theta_{12} \Delta m_{32}^2$$



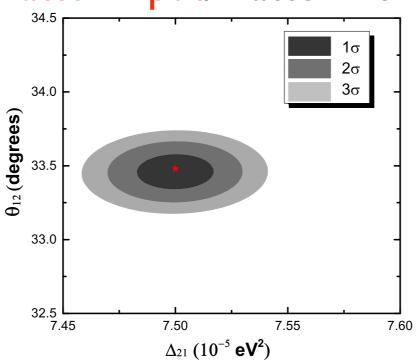
Matter Effects in JUNO



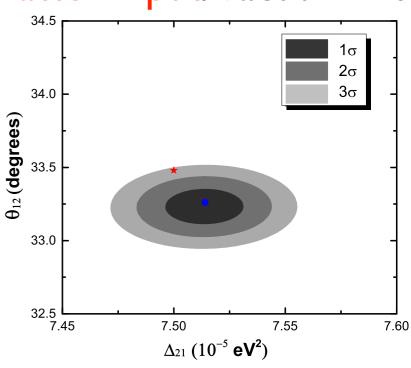
Li, Wang, Xing 1605.00900



Matter Input/Matter Fit



Matter Input/Vacuum Fit



Shift 1σ in Δm_{21}^2 and 3σ in θ_{12}

Size of shift unexplained in 1605.00900:

Khan, Nunokawa, SP upcoming 1810.?? or 1811.??





$u_e ightarrow u_e$ and $ar{ u}_e ightarrow ar{ u}_e$

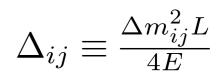
REACTOR NEUTRINOS:

kinematic phase:

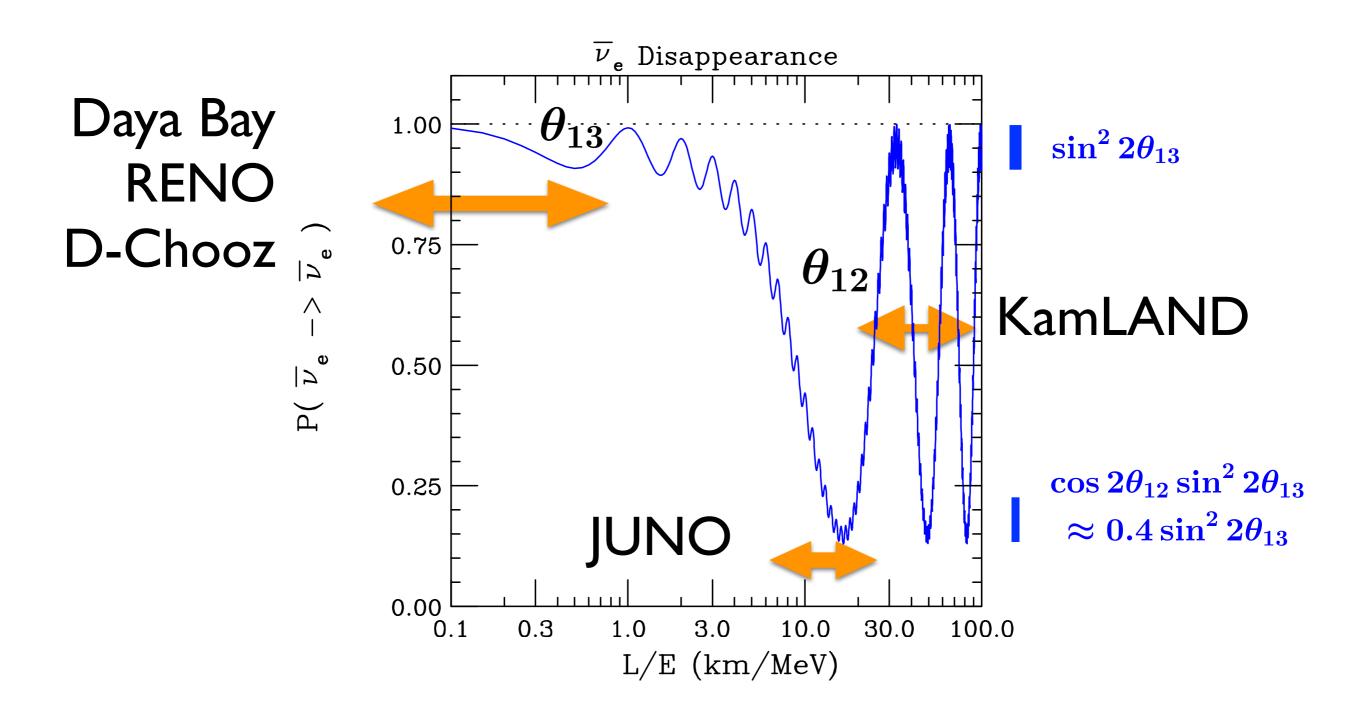
$$\Delta_{ij} \equiv \frac{\Delta m_{ij}^2 L}{4E}$$













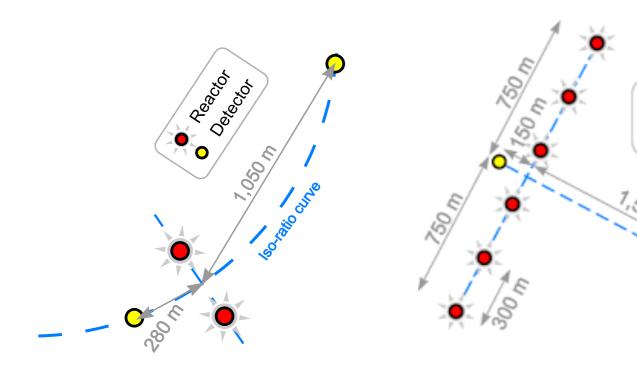


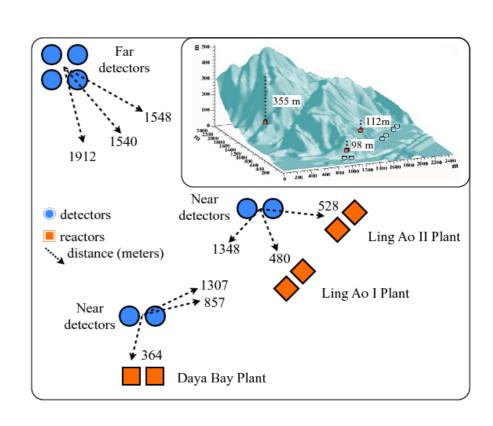
31

Reactor θ₁₃ **Experiments**

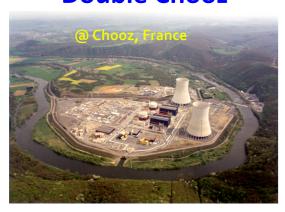
Reactor

Detector





Double Chooz



RENO

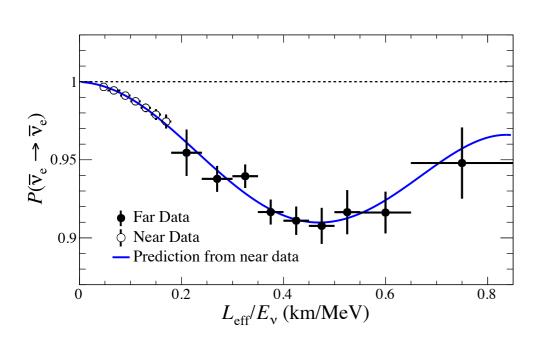


Daya Bay



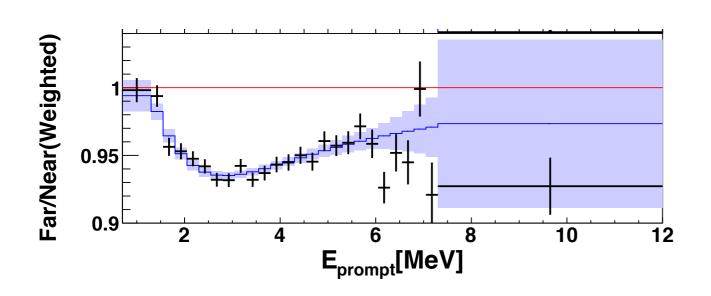


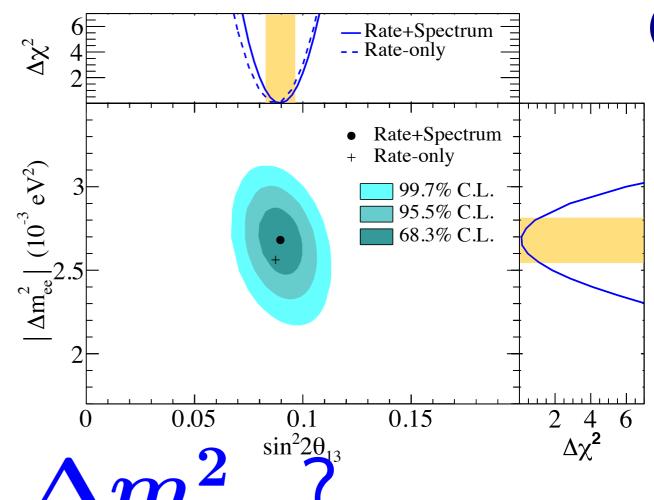
RENO 2200 days



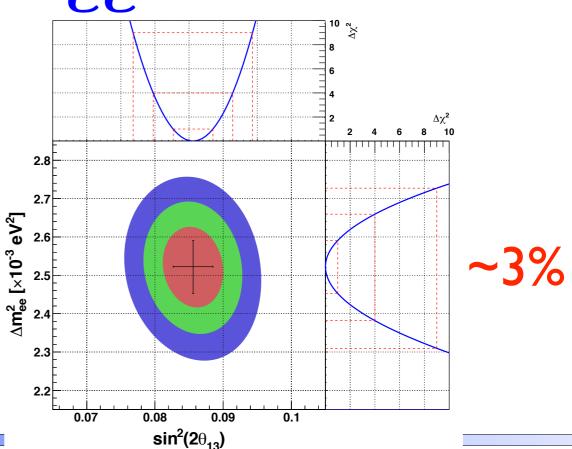
What is

Daya Bay 1958 days





 $\Delta m_{ee}^{2^{- ext{S1}}}$





Survival Probability:



$$\begin{split} P_{ee} &= 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21} \\ &- \sin^2 2\theta_{13} (\cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}) \\ &\approx 1 - (\cos^2 \theta_{13} \sin 2\theta_{12} \Delta_{21})^2 \\ &- \sin^2 2\theta_{13} \sin^2 \Delta_{ee} \end{split}$$

$$\sin^2 \Delta_{ee} \approx \cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}$$

What makes a good Δm_{ee}^2 ?

- ullet good approx. for L/E $< 1 \ km/MeV$
- ullet Simply related to Δm^2_{31} and Δm^2_{32}
- Independent of L/E or "proper age" of the neutrino



" Δm_{ee}^2 Smorgasbord"



$$\Delta m_{ee}^2(\text{DB1}) \equiv \left(\frac{4E}{L}\right) \arcsin \sqrt{\cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta m_{32}^2}$$

exact

1310.6732, 1505.03456v1

$$\Delta m_{ee}^2(\text{DB2}) \equiv \Delta m_{32}^2 + \left(\frac{2E}{L}\right) \arctan\left(\frac{\sin 2\Delta_{21}}{\cos 2\Delta_{21} + \tan^2 \theta_{12}}\right)$$

1505.03456v2,1809.02261

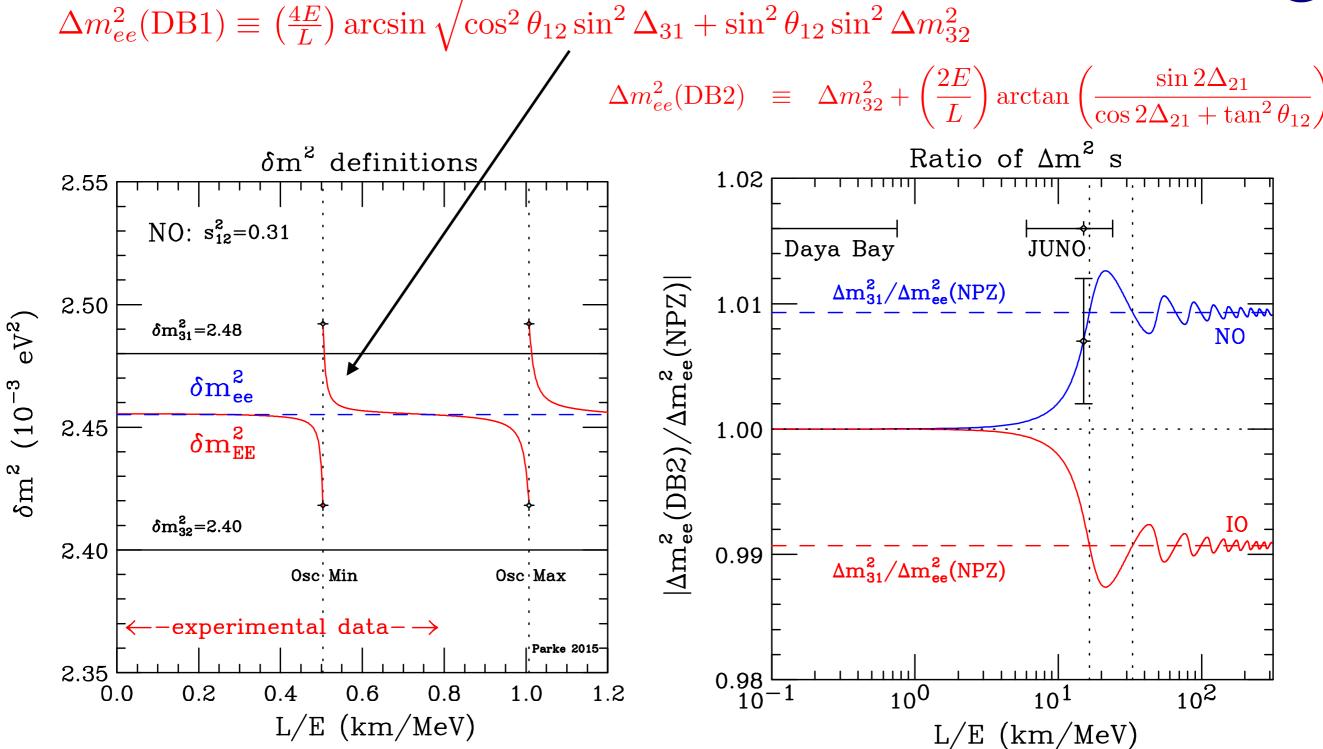
$$\Delta m_{ee}^2(\text{SP}) \equiv \sqrt{\cos^2 \theta_{12} (\Delta m_{31}^2)^2 + \sin^2 \theta_{12} (\Delta m_{32}^2)^2} \approx \Delta m_{ee}^2(\text{NPZ}) \left[1 + \frac{1}{2} s_{12}^2 c_{12}^2 \left(\frac{\Delta m_{21}^2}{\Delta m_{ee}^2} \right)^2 \right]$$

$$10^{-4}$$
1601.074

1601.07464







1310.6732, 1505.03456v1

1903.00148



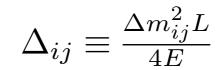


Can Short Baseline Reactor Neutrinos say anything about

$$\Delta m^2_{21}$$

S.H. Seo and SP arXiv:1808.09150



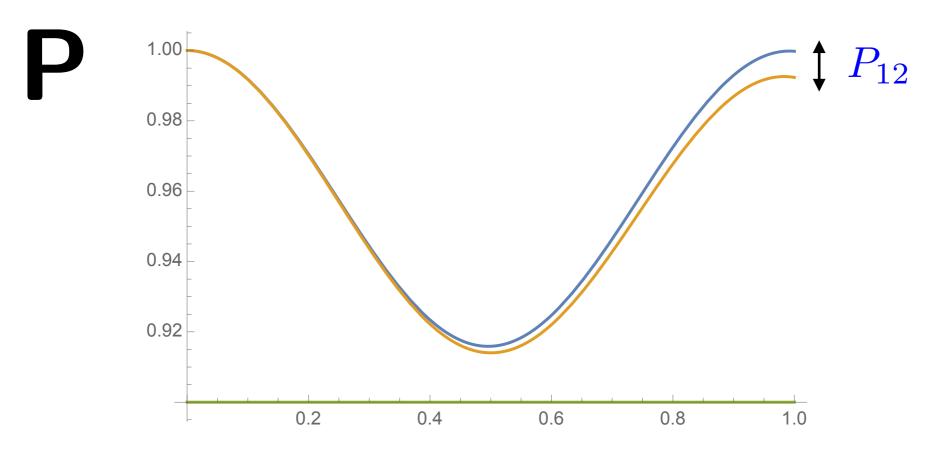




$$P_{ee} = 1 - P_{13} - P_{12}$$

$$P_{13} = \sin^2 2\theta_{13} \sin^2 \Delta_{ee} \qquad (< 0.1)$$

$$P_{12} = (\cos^2 \theta_{13} \sin 2\theta_{12} \Delta_{21})^2 \qquad (< 0.01)$$

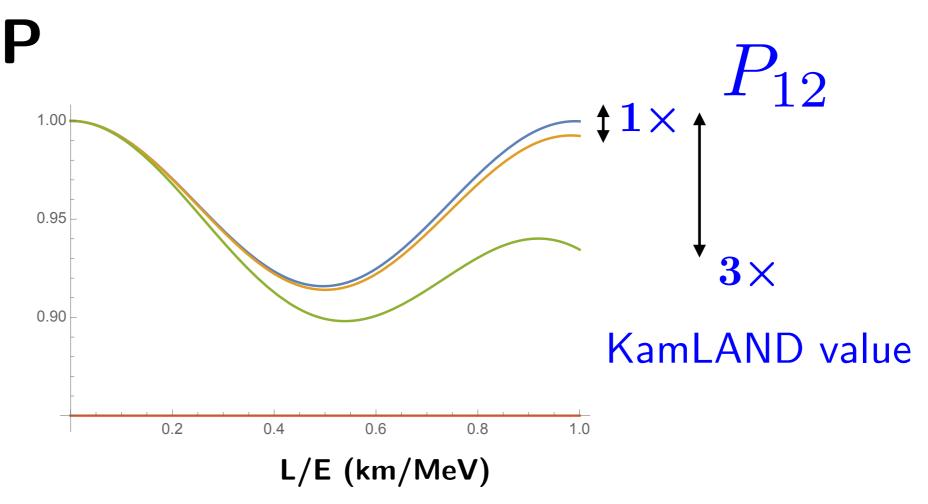


L/E (km/MeV)



Dependence on Solar Parameters:





$$P_{13} \approx 0.08 \sin^2 \left(\frac{\pi}{2} \left(\frac{L/E}{0.5 \,\mathrm{km/MeV}} \right) \right)$$

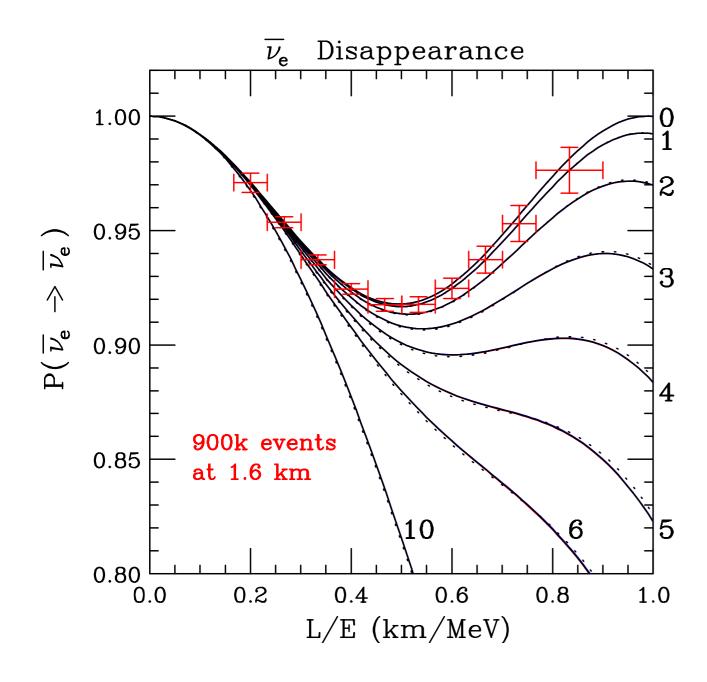
$$P_{12} \approx 0.002 \left(\frac{L/E}{0.5 \,\mathrm{km/MeV}} \right)^2 \left(\frac{\Delta m_{21}^2}{7.5 \times 10^{-5} \,\mathrm{eV}^2} \right)^2$$

If Δm^2_{21} is 3 times bigger, P_{12} is 9 times larger!

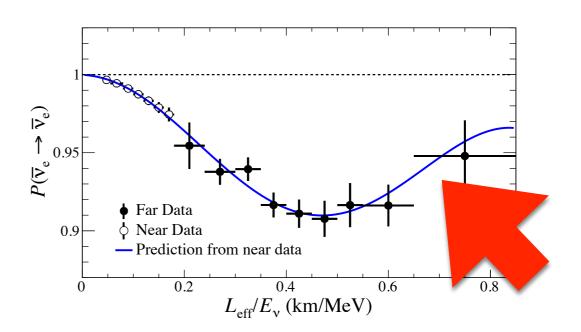


RENO

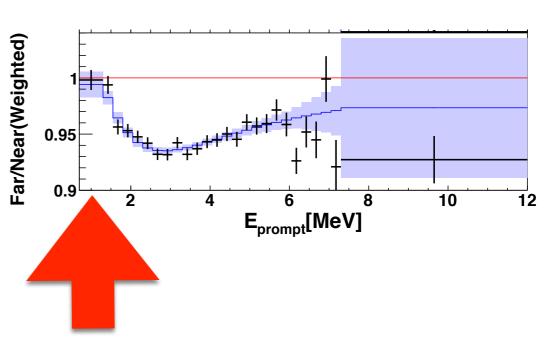




I - I0 KamLAND Δm^2_{21}





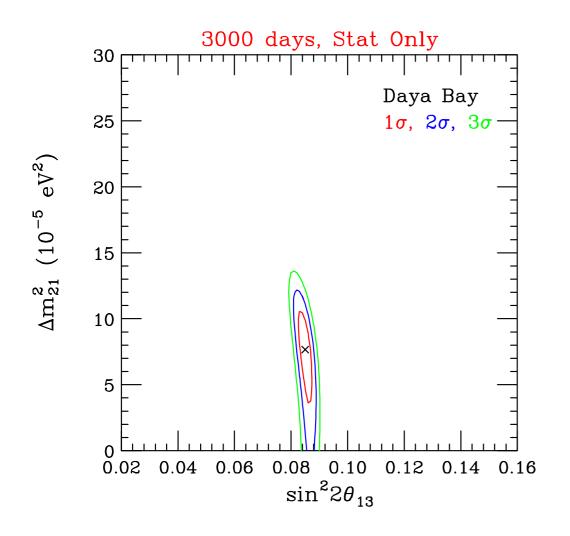


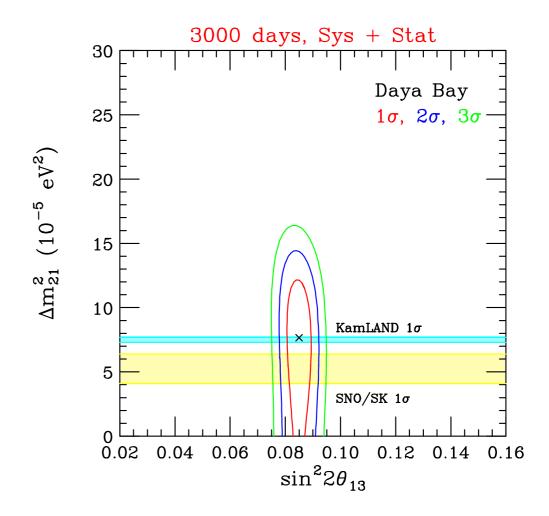
39



Simulation







 $L/E \sim 0.5$ km/MeV compared to KamLAND $L/E \sim 50$ km/MeV

40

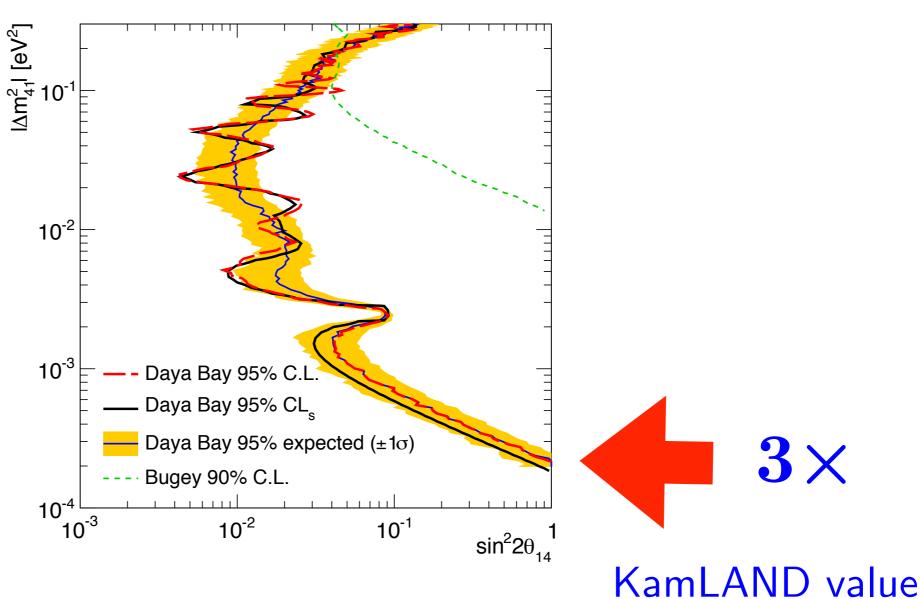




Daya Bay Sterile Neutrino Search

$$\approx 1 - \sin^2 2\theta_{14} \sin^2 \Delta_{41} - \sin^2 2\theta_{13} \sin^2 \Delta_{31}$$
.

1607.01174 404 days



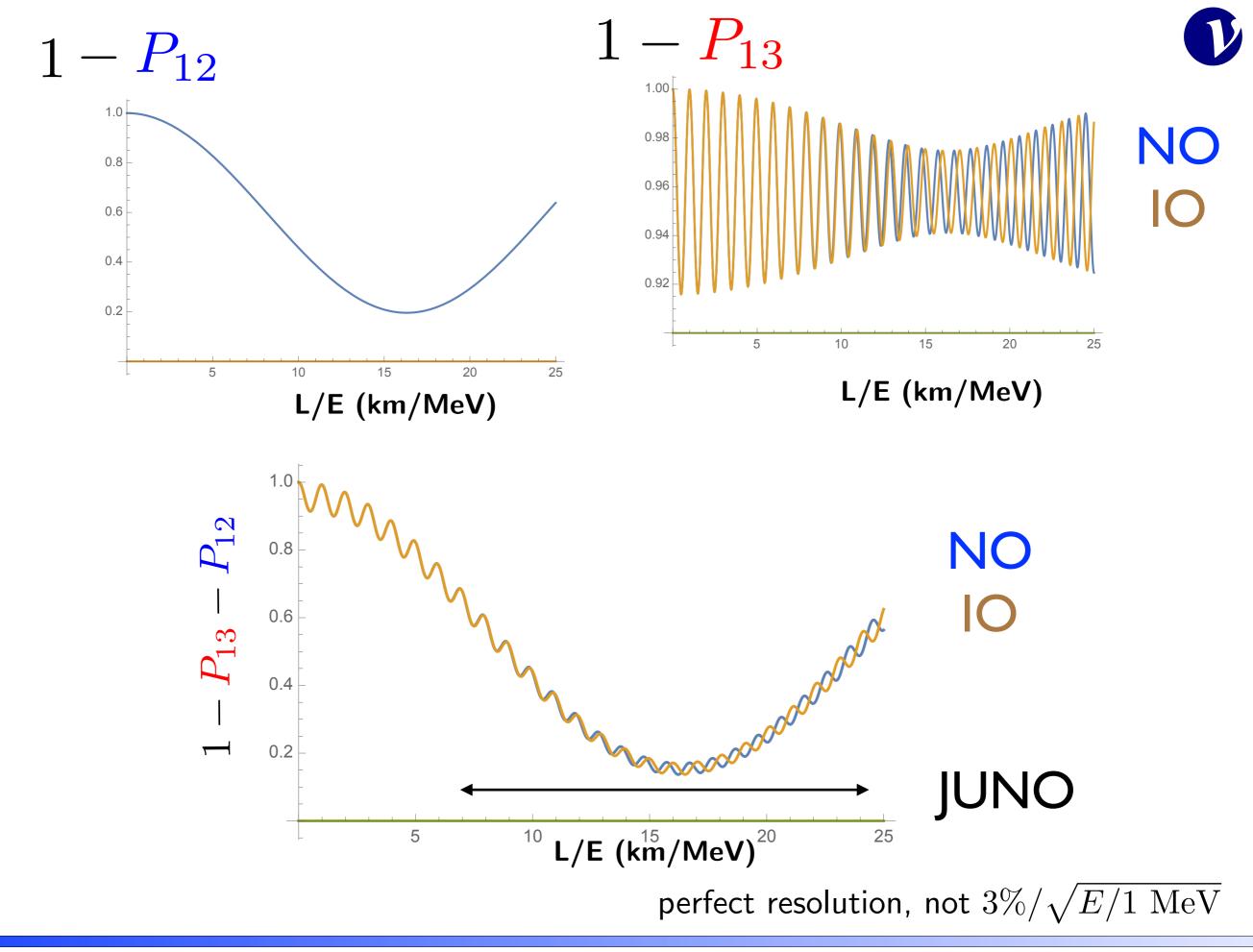
Reinterpretation!





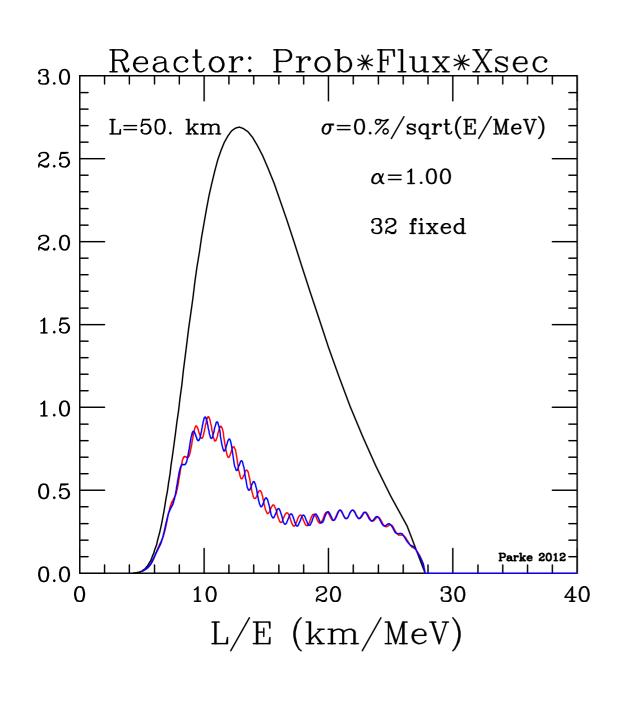
JUNO and the Mass Ordering:

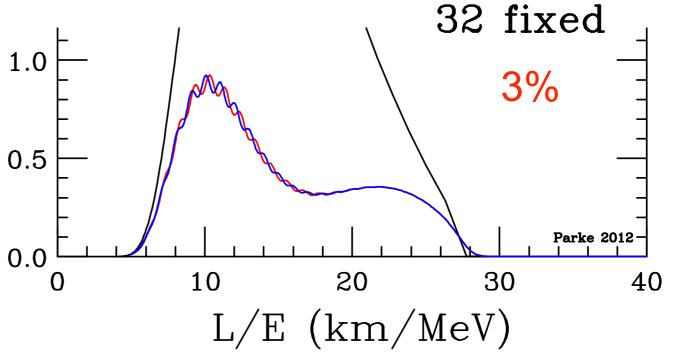


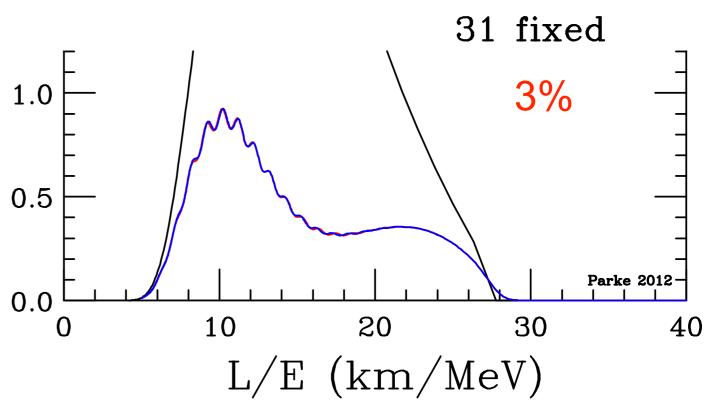






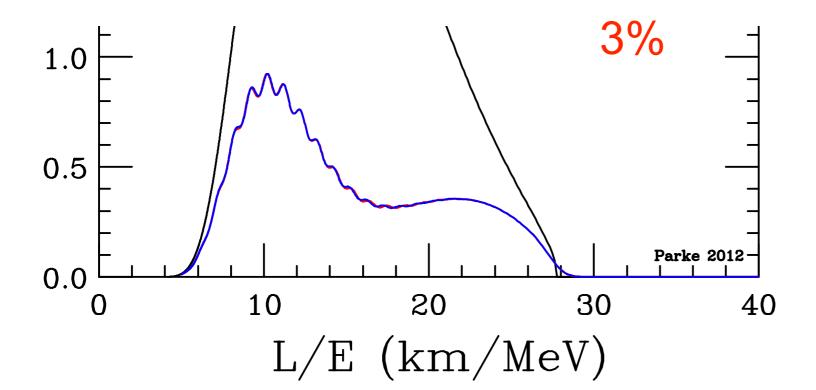


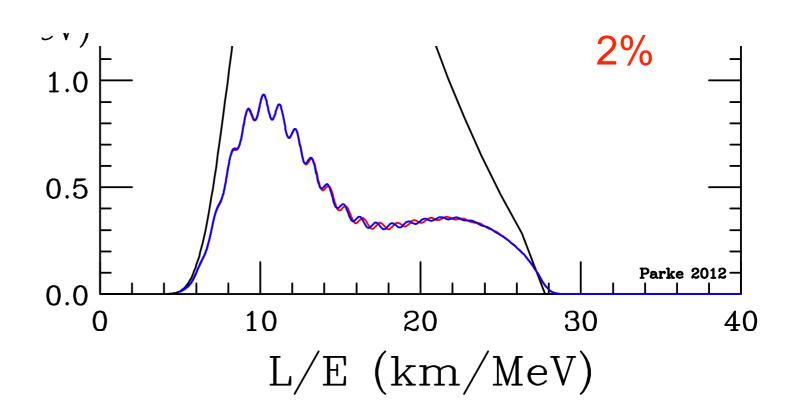








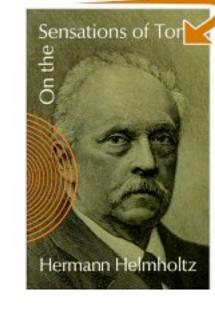






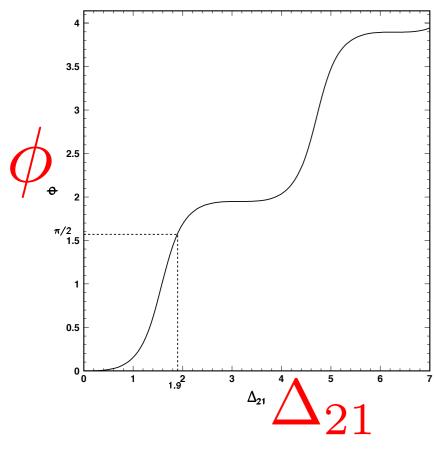
$$P_x(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21}$$

$$-\frac{1}{2} \sin^2 2\theta_{13} \left(1 - \sqrt{1 - \sin^2 2\theta_{12} \sin^2 \Delta_{21}} \cos \Omega \right)$$
with $\Omega = (\Delta_{31} + \Delta_{32}) + \arctan(\cos 2\theta_{12} \tan \Delta_{21}).$



1875

$$\Omega = 2|\Delta_{ee}| \pm \phi$$



$$\phi(\Delta_{21} \pm \pi) = \phi(\Delta_{21}) \pm 2\pi \sin^2 \theta_{12},$$

$$\Delta m_{ee}^2 \equiv \frac{\partial \Omega}{\partial (L/2E)} \Big|_{\frac{L}{E} \to 0} = \cos^2 \theta_{12} \Delta m_{31}^2 + \sin^2 \theta_{12} \Delta m_{32}^2$$

$$\phi = \left\{ \arctan(\cos 2\theta_{12} \tan \Delta_{21}) - \Delta_{21} \cos 2\theta_{12} \right\}$$

NO phase advance
 IO phase retardation





$$\Omega = (\Delta_{31} + \Delta_{32}) + \arctan(\cos 2\theta_{21} \tan \Delta_{21}) \quad \text{NPZ}$$

$$= 2\Delta_{ee} + \arctan(\cos 2\theta_{12} \tan \Delta_{21}) - \Delta_{21} \cos 2\theta_{12}$$

$$= 2\Delta_{32} + \arctan\left(\frac{\sin 2\Delta_{21}}{\cos 2\Delta_{21} + \tan^2 \theta_{12}}\right) \quad \text{DB}$$

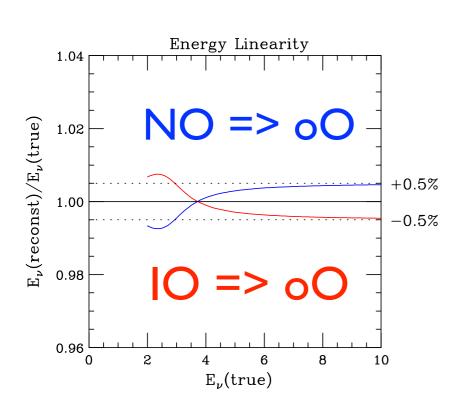
$$= 2\Delta_{31} - \arctan\left(\frac{\sin 2\Delta_{21}}{\cos 2\Delta_{21} + \cot^2 \theta_{12}}\right)$$



Neutrino Energy Reconstruction



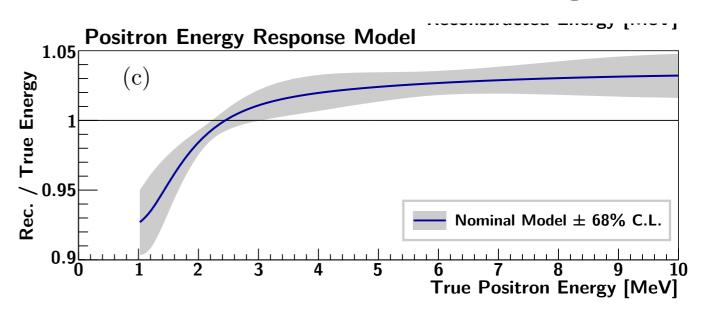
RENC



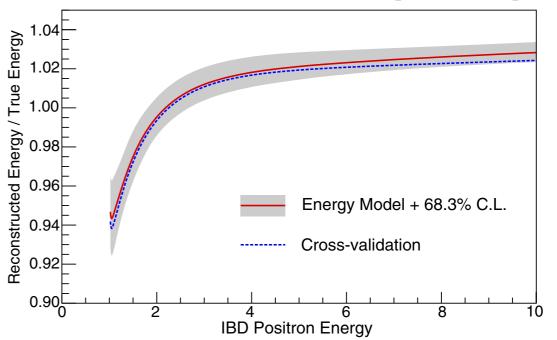
$$\Omega = 2|\Delta_{ee}| + \eta \phi$$

(NO, oO, IO) given by $\eta = (1, 0, -1)$

$$\delta~(\Delta m_{ee}^2) \sim 0.5\%$$



Daya Bay



Parke @ NOW 2008

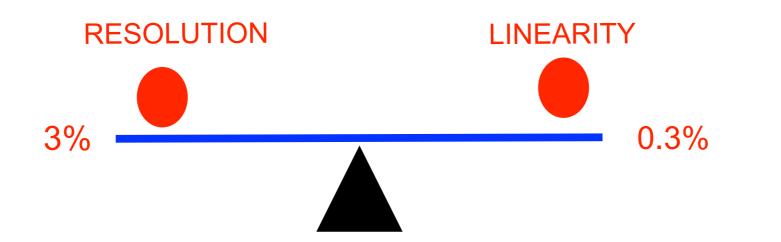




- Energy Resolution to 3% or lower at 1 MeV
- Linearity to sub 1% precision for the reconstructed neutrino energy

	KamLAND	JUNO	RENO-50
LS mass	~1 kt	20 kt	18 kt
Energy Resolution	6%/	~3%/	~3%/
Light yield	250 p.e./MeV	1200 p.e./MeV	>1000 p.e./MeV

Linearity 1.9% < 0.5% < 0.5% > 4 x







Matter Effects:





Neutrino Propagation in Matter:

$$i\frac{d}{dx}\nu = H\nu$$
 $\nu \equiv \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$

$$H = \frac{1}{2E} \left\{ U \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{bmatrix} U^{\dagger} + \begin{bmatrix} a(x) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}$$

$$a = 2\sqrt{2}G_F N_e E \approx 1.52 \times 10^{-4} \left(\frac{Y_e \rho}{\text{g.cm}^{-3}}\right) \left(\frac{E}{\text{GeV}}\right) \text{eV}^2.$$

if $ho Y_e = 1.5$ g/cm 3 and E = 10 GeV then $a pprox \Delta m_{31}^2$

$$E=300~MeV$$
 then $approx \Delta m_{21}^2$



Neutrino Evolution in Matter (conti):



$$U_{23}^{\dagger}(\theta_{23}, \delta) H U_{23}(\theta_{23}, \delta) = H_D + H_{OD}$$

D=diagonal OD= off-diagonal

$$(2E) H_D = \begin{bmatrix} a + s_{13}^2 \Delta m_{ee}^2 \\ (c_{12}^2 - s_{12}^2) \Delta m_{21}^2 \\ c_{13}^2 \Delta m_{ee}^2 \end{bmatrix}$$

 $\Delta m_{ee}^2 \equiv \cos^2 \theta_{12} \Delta m_{31}^2 + \sin^2 \theta_{12} \Delta m_{32}^2$

!!! level crossing !!!

$$(2E) H_{OD}/\Delta m_{ee}^{2} = s_{13}c_{13} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$0.15 + c_{13} s_{12}c_{12} \left(\frac{\Delta m_{21}^{2}}{\Delta m_{ee}^{2}}\right) \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$- s_{13} s_{12}c_{12} \left(\frac{\Delta m_{21}^{2}}{\Delta m_{ee}^{2}}\right) \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$0.002$$



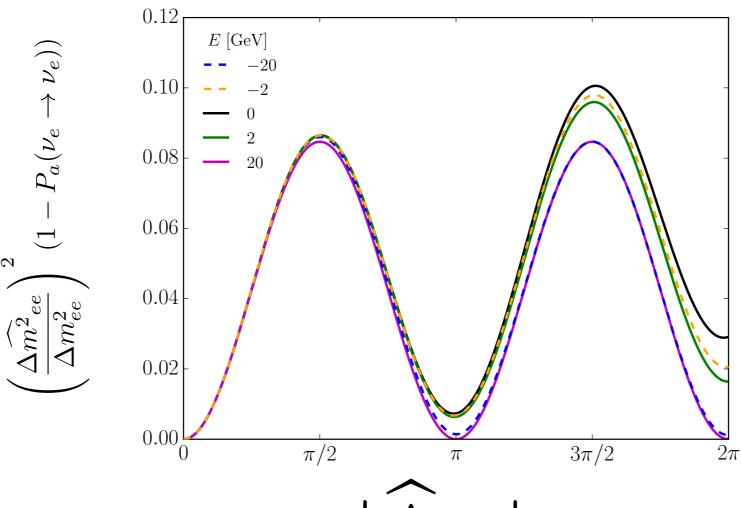




$$u_e \rightarrow
u_e$$

$$P_a(\nu_e \rightarrow \nu_e) \approx 1 - \sin^2 2\theta_{13} \left(\frac{\Delta m_{ee}^2}{\Delta \widehat{m}^2_{ee}}\right)^2 \sin^2 \widehat{\Delta}_{ee}, \qquad \widehat{\Delta}_{ee} \equiv \Delta \widehat{m}^2_{ee} L/(4E),$$

$$\widehat{\Delta m^2}_{ee} \approx \Delta m_{ee}^2 \sqrt{(\cos 2\theta_{13} - a/\Delta m_{ee}^2)^2 + \sin^2 2\theta_{13}},$$



Denton, SP 1808.09453

$$|\widehat{\Delta}_{ee}|$$



In Matter:

$$\widehat{J} pprox rac{J}{\mathcal{S}_{\odot}\,\mathcal{S}_{
m atm}}\,,$$



two, two flavor resonance factors:

$$\mathcal{S}_{\odot} = \sqrt{(\cos 2 heta_{12} - c_{13}^2 a/\Delta m_{21}^2)^2 + \sin^2 2 heta_{12}} \,,$$
 2 or I % effects !

$$S_{\text{atm}} = \sqrt{(\cos 2\theta_{13} - a/\Delta m_{ee}^2)^2 + \sin^2 2\theta_{13}}$$
.

Denton & SP 1902.07185

fractional difference:
$$\sin^2\theta_{13} \left(\frac{\Delta m_{21}^2}{\Delta m_{ee}^2}\right) \cos 2\theta_{12} \sim 0.04\%$$



Summary:



- Observation of Solar Neutrinos and Reactor Neutrinos have taught us a great deal the electron row of the PMNS matrix, about U_ei and Delta m^2 's.
- the concept of an effective Delta m^2, Delta m^2_ee, is useful for the shape analysis of reactor neutrinos.

$$\Delta m_{ee}^2$$
 is ν_e average of Δm_{31}^2 and Δm_{32}^2

- Short baseline reactor experiments can constrain (maybe measure) Delta m^2_21 at twice the KamLAND value.
- the generalization of Delta m^2_ee into matter, is useful for understanding neutrino oscillations in matter for DUNE and T2HK(K)

$$\Delta m_{ee}^2 \sqrt{(\cos 2 heta_{13} - a/\Delta m_{ee}^2)^2 + \sin^2 2 heta_{13}}$$



Ernest Rutherford, master of simplicity

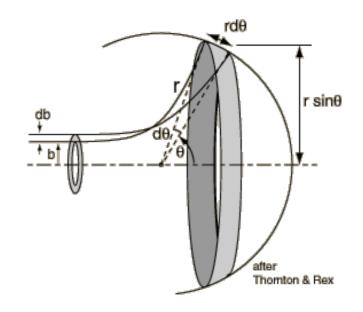


By Ashutosh Jogalekar | August 30, 2013 |



Ernest Rutherford, emperor of the atomic domain (Image: Wikipedia Commons)

"theorists play games with their symbols while we discover truths about the universe". And yet



$$\sigma = \pi Z^2 \left(\frac{ke^2}{KE}\right)^2 \left(\frac{1 + \cos\theta}{1 - \cos\theta}\right)$$

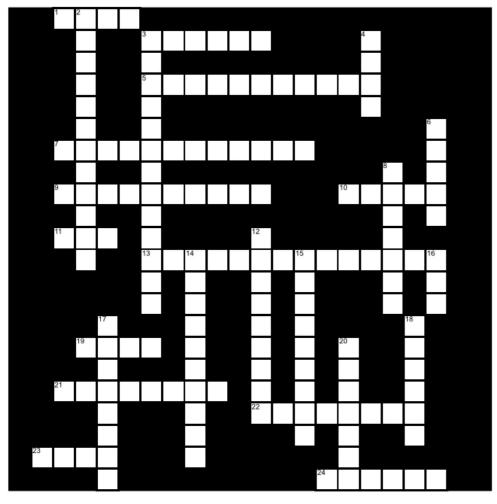
he had an eye for theoretical talent that allowed him to nurture Niels Bohr, as dyed-in-the-wool a theoretician and philosopher as you could find.





Neutrino Crossword





Across

- 1 When Potassium 40 decays does it emit neutrinos or antineutrinos?
- 3 In 1966 a popular book on neutrinos was written by
- **5** How many neutrinos, in log base 10, does the Sun emit per second?
- 7 What important effect did Wolfenstein discover in 1978 ?
- **9** What percentage of the energy from a Supernova is released in neutrinos?
- 10 Neutrinos from Decay of this element have been observed
- 11 Solar Neutrino Unit
- 13 Why are neutrino nucleon cross sections so challenging to calculate?
- 19 Neutrino Propagation states
- 21 What distinguishes a neutrino from and antineutrino?
- 22 Little neutral one
- 23 What happens to oscillation length if Planck's constant goes to zero?
- 24 If neutrinos are Majorana which number symmetry is violated ?

Down

- 2 Quantum mechanical interference of the mass eigenstate leads to ...
- 3 What do reactors emit?
- **4** Why Pauli did not go to the scientific meeting where his invention of the neutrino was announced?
- 6 When crossing a high energy neutrino beam is it better to cross in front or behind a concrete wall?
- 8 Powers Nuclear Reactors
- **12** Who gave the SuperK atmospheric neutrino talk at Neutrino 1998
- **14** Why is |U_e1|^2 larger than |U_e2|^2 or |U_e3|^2?
- **15** The Argon in earth's atmosphere comes from decay of which element?
- 16 Which experiment "nailed" the solar neutrino anomaly?
- 17 The invariant that controls the size of CP violation was invented/discovered by this woman physicist
- 18 Neutrino Interaction States
- 20 Zombie neutrinos

npc.fnal.gov/question/

Neutrino Question · Neutrino Physics Center







Neutrino Question:

You receive an email from a high school student taking Advanced Placement Physics, asking:

"Why are you studying Neutrinos?"

Nu Seminar / Fermilab

58





Trevor Nichols

The purest answer to why we study neutrinos is simply that we are curious. Neutrinos are the most abundant of the fundamental particles, yet we know the least about them. Beyond simple curiosity, neutrinos are useful, not so much for what they can do themselves, but for what they can tell us about other things.

Neutrinos provide a new lens through which to view the universe. Black holes are not detectable using visible light. They do not emit any. Yet, when we look at the universe using X-rays, we can now see swirling accretion discs, indicating the presence of black holes. Similarly, neutrinos provide visibility to that which we previously could not see.

Neutrinos are created in nuclear reactions, which are abundant in the cores of stars. Unlike photons, neutrinos can escape a star (or any other object) unaltered by the material they must pass through to escape. Upon detection, neutrinos reveal information about the originating reaction.





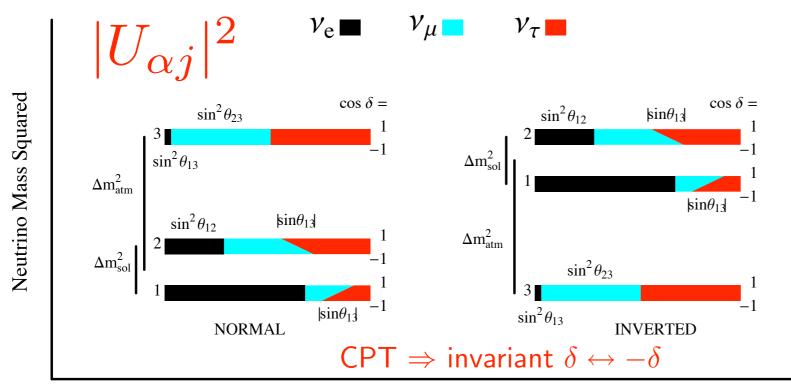
extras



Summary:



• Labeling massive neutrinos: $|U_{e1}|^2 > |U_{e2}|^2 > |U_{e3}|^2$



Fractional Flavor Content varying
$$\cos \delta$$

$$\sin^2 \theta_{12} \sim \frac{1}{3}$$
 $\sin^2 \theta_{23} \sim \frac{1}{2}$
 $\sin^2 \theta_{13} \sim 0.02$

$$0 \leq \delta < 2\pi$$

$$|\Delta m_{21}^2| = |m_2^2 - m_1^2| = 7.5 \times 10^{-5} \text{ eV}^2$$
 $L/E = 15 \text{ km/MeV} = 15,000 \text{ km/GeV}$ $|\Delta m_{31}^2| = |m_3^2 - m_1^2| = 2.5 \times 10^{-3} \text{ eV}^2$ $L/E = 0.5 \text{ km/MeV} = 500 \text{ km/GeV}$





How Does Daya Bay Define Δm_{EE}^2 ?

arXiv:1310.6732

 ν_j and ν_i . Since $\Delta m_{21}^2 \ll \left| \Delta m_{31}^2 \right| \approx \left| \Delta m_{32}^2 \right|$ [1], the short-distance (\sim km) reactor $\overline{\nu}_e$ oscillation is due primarily to the Δ_{3i} terms and naturally leads to the definition of the effective mass-squared difference $\sin^2 \Delta_{ee} \equiv \cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}$ [11].

1505.03456v1

[8] $\sin^2 \Delta_{ee} \equiv \cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}$, where $\Delta_{ji} \equiv 1.267 \Delta m_{ji}^2 (\text{eV}^2) [L(\text{m})/E(\text{MeV})]$, and Δm_{ji}^2 is the difference between the mass-squares of the mass eigenstates ν_j and ν_i .

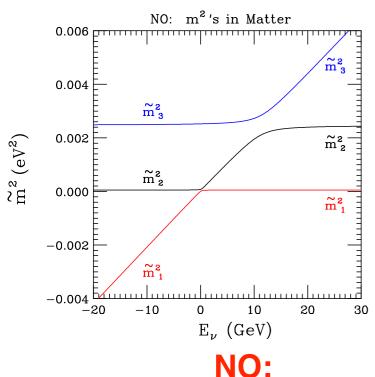
62





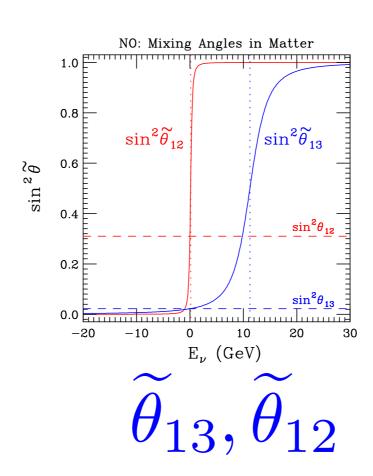


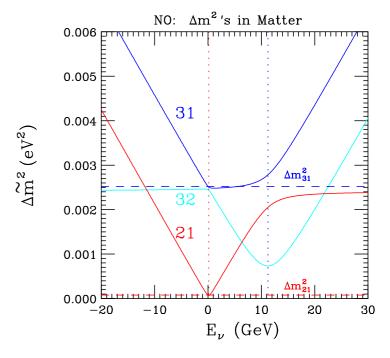
0th Order:



for IO figures see 1604.08167

$heta_{23}, \delta$ unchanged





$$\Delta \, \widetilde{m^2}_{31}, \Delta \, \widetilde{m^2}_{21}$$

After 2 rotations:

$$(2E) H_{OD}/\Delta m_{ee}^{2} = \sin(\tilde{\theta}_{13} - \theta_{13}) s_{12} c_{12} \left(\frac{\Delta m_{21}^{2}}{\Delta m_{ee}^{2}}\right) \begin{bmatrix} & -\tilde{s}_{12} \\ & \tilde{c}_{12} \end{bmatrix}$$

 4×10^{-4}

$$\sin(\widetilde{\theta}_{13} - \theta_{13}) \approx s_{13}c_{13} \left(\frac{a}{\Delta m_{ee}^2}\right)$$

zero in vacuum

for
$$E=2~{\rm GeV}$$
 and $\rho=3~{\rm g.cm^{-3}}$

 $\widetilde{s}_{12} \equiv \sin \widetilde{ heta}_{12}$, etc